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Effects of rotation and feedback control on Bénard–Marangoni convection

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ABSTRACT

The effect of a feedback control on the onset of steady and oscillatory Bénard–Marangoni instability in a rotating horizontal fluid layer is considered theoretically using linear stability theory. It is demonstrated that generally the critical Marangoni number for transition from the no-motion (conduction) to the motion state can be drastically increased by the combined effects of feedback control and rotation. The role of the controller gain parameter on the $Pr - Ta$ and $Pr - R/R_c$ parameter spaces, dividing stability domains into which either steady or oscillatory convection is preferred, is determined.

1. Introduction

Bénard convection, sometimes referred to as Bénard–Marangoni convection, was first observed by Henri Bénard [1]. Bénard convection occurs when a horizontal layer of fluid is heated uniformly from below, which causes the heated fluid to rise because of local density differences. The warm fluid near the bottom is replaced by cooler fluid near the top. If the thickness of the fluid is small in comparison to the expanse of its surface, the fluid will tend to circulate in a series of cells known as Bénard cells. The instability of Bénard–Marangoni convection is due to the combined effects of thermal buoyancy and surface tension.

The instability of the convection driven by buoyancy is referred to as Rayleigh-Bénard instability. It was studied by Chandrasekhar [2] and Drazin and Reid [3]. Another effect is due to local variation in surface tension. This type of convective instability is referred to as Marangoni instability and was first theoretically analysed by Pearson [4]. The effect of the surface deflection on Marangoni instability was later considered by Scriven and Sternling [5]. As these two kinds of instability take place at the same time, the instability mechanism is known as the Bénard–Marangoni instability. Nield [6] first analysed the Bénard–Marangoni instability problem. Davis and Homsy [7] later studied the effect of surface deflection on the combined Bénard–Marangoni effect. Stability analysis of Bénard–Marangoni convection of a low Prandtl number fluid in an open vertical cylinder was studied by Xu et al. [8]. Medale and Cerisier [9] investigate numerical simulation of Bénard–Marangoni convection of fluid layer heated from below in small aspect ratio container. A numerical study of the relative importance of Marangoni effects under microgravity conditions is presented by Giangi et al. [10]. A top free liquid layer is heated from the bottom in a two-dimensional rectangular container with insulated side walls was studied by [11].

Pérez-Garcia and Carneiro [12] have carried out a systematic study of the linear stability of Bénard–Marangoni convection with a deformable free surface. In their study, convective instability was induced by the temperature gradient which decreased linearly with liquid layer height. Sparrow et al. [13] and Roberts [14] analysed thermal instability in a horizontal fluid layer with the nonlinear temperature distribution which is created by internal heat generation. Boeck and Thess [15] studied Bénard–Marangoni convection at low Prandtl number with periodic boundary condition in both horizontal directions and either a free-slip or no-slip bottom wall of the two-dimensional case. Gasser and Kazimi [16] and Kaviany [17] investigated the effect of the internal heat generation on the onset of convection in a porous medium. Very recently, Idris et al. [18] studied the effect of a cubic temperature profile on the onset of steady Bénard–Marangoni convection in a micropolar fluid.

The ability to control complex convective flow patterns is important in both technology and fundamental science. In many technological processes, the naturally occurring flow patterns may not be the optimal ones. By controlling the flow, one may be able to optimize the process, improve product quality, and achieve significant savings. The ability to stabilise otherwise nonstable states may also assist one in gaining deeper insights into the dynamics of flows. Delaying the onset of convection by the use of linear and nonlinear control strategies was described by Tang and Bau [19,20] and Bau [21]. Bau [21] extended the studies

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Nomenclature

a	total horizontal wave number
Bi	Biot number
Bo	Bond number
C_g	Control parameter
Cr	Crispation number
d	initial thickness of the layer
g	gravitational acceleration
k	thermal conductivity
M	Marangoni number
Pr	Prandtl number
R	Rayleigh number
s	temporal growth rate
Ta	Taylor number
$\theta(z)$	vertical variation of temperature perturbation
$W(z)$	vertical variation of vertical velocity perturbation
$K(z)$	vertical variation of vertical vorticity perturbation
x, y, z	spatial Cartesian coordinates

Greek symbols

γ	coefficient of surface tension
κ	thermal diffusivity
μ	dynamic viscosity of fluid
ν	kinematic viscosity of fluid
ρ	density of fluid
σ	growth rate
ω	frequency
Ω	angular velocity
ΔT	temperature difference

Subscript

c	critical state

Superscripts

-	basic state
′	perturbed state

of Pearson [4] and Takashima [22,23] by including a feedback control strategy effecting small perturbations in boundary data to suppress the onset of Marangoni convection. Or et al. [24] employed a nonlinear feedback control strategy to delay the onset and eliminate the subcritical long-wavelength instability of Marangoni–Bénard convection. Or and Kelly [25] showed that the weakly nonlinear flow properties in the Rayleigh–Bénard–Marangoni problem can be altered by linear and nonlinear proportional feedback control processes and the stabilization of the basic state can be achieved. Remillieux et al. [26] delineated the mechanism that lead to oscillatory Rayleigh-Bénard convection in the presence of large controller gains and the application of derivative controller to suppress oscillatory convection. Recently, Hashim and Siri [27] and Siri and Hashim [28,29] applied Bau's [21] feedback control strategy to Marangoni instability in a rotating fluid layer. Bau's [21] control strategy has also been applied by Hashim and Awang-Kechil [30] to delay the onset of Marangoni convection in variable viscosity fluids.

In this work, we use classical linear stability analysis to obtain the thresholds and codimension-2 points for the onset of steady and oscillatory convection in a rotating fluid layer in the presence of a feedback control strategy. We are concerned with the effect of the feedback control on the Bénard–Marangoni instability of a horizontal liquid layer with a non-deformable upper free surface.

2. Mathematical formulation

In this study, the stability of a horizontal layer of quiescent fluid of thickness d which is unbounded in the horizontal x- and y-directions (see Fig. 1). The layer is kept rotating uniformly around the vertical z-axis with a constant angular velocity Ω. The surface tension γ takes the form $\gamma = \gamma_0 - \tau(T - T_0)$, where γ_0 and T_0 are reference values, and τ is the rate of change of surface tension with the temperature. The upper free surface is assumed non-deformable. In the reference state, the fluid is at rest with respect to the rotating axes and heat is conducted between the lower boundary ($z = 0$) and the interface. Following Koschmieder [31], when motion sets in, the velocity $\mathbf{v} = (u, v, w)$, pressure p and temperature T fields obey the usual balance equations of mass, momentum and energy,

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} + 2\Omega \times \mathbf{v} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{v} + \mathbf{g}, \tag{2}$$

$$\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla)T = \kappa\nabla^2 T, \tag{3}$$

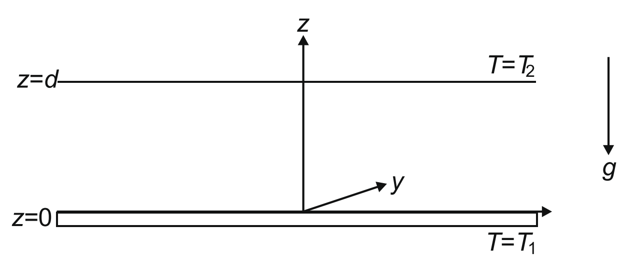

Fig. 1. Sketch of problem geometry.

where ρ is the fluid density, ν is the kinematic viscosity, κ is the thermal diffusivity and $\mathbf{g} = (0, 0, -g)$ is the gravitational field. At the free deformable surface, the boundary conditions comprise of kinematic, the heat flux, the two shear stress and the normal stress conditions which are given by, respectively,

$$w = 0, \tag{4}$$

$$k\nabla T \cdot \mathbf{n} + hT = 0, \tag{5}$$

$$2\mu D_{\mathrm{nt}} = \frac{\partial \gamma}{\partial T}\nabla T \cdot \mathbf{t}, \tag{6}$$

$$(p_a - p) + 2\mu D_{\mathrm{nn}} = \gamma\nabla \cdot \mathbf{n}, \tag{7}$$

where h is the heat transfer coefficient, k is the thermal conductivity of the fluid, μ is the dynamic viscosity, p_a is the pressure of the atmosphere, D_{ij} is the rate of strain tensor, and $\mathbf{t}$ and $\mathbf{n}$ denote tangential and normal unit vectors, respectively.

At the lower boundary we have the condition of continuity of velocity between the solid and the fluid. This lower boundary is assumed to be isothermal. We introduce infinitesimal disturbances to the governing equations and boundary conditions by setting

$$(u, v, w, \rho, p, T) = (0, 0, 0, \bar{\rho}, \bar{p}, \overline{T}) + (u', v', w', \rho', p', \theta'), \tag{8}$$

where the quantities with bars represent basic states and primed quantities represent perturbed variables written in normal mode forms

$$[w', \theta', \zeta'] = [W(z), \Theta(z), K(z)]e^{i(a_x x + a_y y) + \tilde{s}t}, \tag{9}$$

where a_x and a_y are wavenumbers of disturbances in the x- and y-directions, respectively, and ζ' is the dimensionless pertubed vorticity in the z-direction. W, Θ and K are amplitudes of vertical velocity, temperature and vertical vorticity, respectively. The growth parameter $\tilde{s}$ is in general complex and denoted by $\tilde{s} = \sigma + i\omega$, where σ is the growth rate of the instability and ω is the frequency. If $\sigma > 0$, the disturbances grow and the system becomes unstable. If $\sigma < 0$, the disturbances decay and the system becomes stable. When $\sigma = 0$, the instability of the system at the marginal state, sets in as stationary motions, provided $\omega = 0$, or an oscillatory motions, provided $\omega \neq 0$.

In the proportional feedback control of Bau [21], the actuators are placed at the bottom heated surface. In addition, sensors are used to detect the departure of the surface temperature from its conductive state. Following Bau [21], the determination of a control, $q(t)$, can be accomplished using the proportional-integral-differential (PID) controller of the form

$$q(t) = r + C_g[e(t)], \quad e(t) = \hat{m}(t) - m(t), \tag{10}$$

where r is the calibration of the control, $e(t)$ an error or deviation from the state measurement, $\hat{m}(t)$, from some desired or reference value, $m(t), C_g = C_P + C_D \mathrm{d}/\mathrm{d}t + C_I \int_0^t \mathrm{d}t$ with C_P is the proportional gain, C_D differential gain and C_I integral gain. Based on (10), for one sensor plane and proportional feedback control, the actuator modifies the heated surface temperature using a proportional relationship between the upper $(z = z_u)$ and lower $(z = z_l)$ thermal boundaries for the perturbation field,

$$T(x, y, z_l, t) - T(x, y, z_l) = -C_g[T(x, y, z_u, t) - T(x, y, z_u)], \tag{11}$$

or equivalently,

$$T'(x, y, z_l, t) = -C_g T'(x, y, z_u, t), \tag{12}$$

where T' denotes the deviation of the fluid's temperature from its conductive value.

3. The linearised problem

Following the classical lines of linear stability theory as presented in [2], the linearised and dimensionless governing equations can be written as

$$(D^2 - a^2)(D^2 - a^2 - s)W = Ta^* DK + a^2 R\Theta, \tag{13}$$

$$(D^2 - a^2 - s)K = -Ta^* DW, \tag{14}$$

$$(D^2 - a^2 - sPr)\Theta = -W. \tag{15}$$

The equations have been written in dimensionless form using d/π, $d^2/\pi^2 \nu$ and $\nu \Delta T/\pi \kappa$ as the scales for distance, time and temperature, respectively, where ν is the kinematic viscosity, κ is the thermal diffusivity and ΔT is the temperature difference between the top and bottom surfaces. The operator $D = \mathrm{d}/\mathrm{d}z$ denotes differentiation with respect to the vertical coordinate z, $a = kd/\pi$ is the dimensionless wavenumber and $s = \tilde{s} d^2/\pi^2 \nu$ is the stability parameter.

At the bounding surfaces, the following conditions must be satisfied:

$$W = 0, \tag{16}$$

$$(D^2 + a^2)W + a^2 M^* \Theta = 0, \tag{17}$$

$$D\Theta + Bi^* \Theta = 0, \tag{18}$$

$$DK = 0, \tag{19}$$

on $z = \pi$, and

$$W = DW = K = 0, \tag{20}$$

on $z = 0$, together with the controlled condition,

$$\Theta(0) + C_g \Theta(\pi) = 0. \tag{21}$$

The starred dimensionless numbers are defined by $M^* = M/\pi^2$, $R^* = R/\pi, Ta^* = Ta/\pi^4, Bi^* = Bi/\pi$ where M, Ta, Bi and Pr are the standard Marangoni, Rayleigh, Taylor, Biot and Prandtl numbers, respectively.

4. Solution approach

Since the solution method is standard, we only give a brief description of the solution approach in this section. Following a similar procedure as employed in Hashim and Sarma [32], combining Eqs. (13)–(15), then gives a single linear eight order ordinary differential equation for Θ. This eight order ordinary differential equation together with the boundary conditions (16)–(21) can be turned into the eigenvalue problem of the form

$$f(M, R, Ta, Pr, Bi, C_g, a, s) = 0, \tag{22}$$

from which the stability domains of the problem can be obtained. The real and imaginary parts of (22) give a system of two nonlinear equations for the eigenvalues s and M. The NAG Fortran routine C05NBF is then used, given values for the other parameters, to find a zero, $\omega = \mathrm{Im}(s(R, Ta, Pr, Bi, C_g, a))$ and $M = M(R, Ta, Pr, Bi, C_g, a)$ with $\sigma = 0$, of the system of nonlinear equations.

5. Results and discussion

In this work, we studied the effect of C_g on the onset of steady and oscillatory Bénard–Marangoni convection in a rotating fluid layer. Also, in this study we describe the results for the case of a non-deformable upper surface and an insulated upper surface, $Bi = 0$. The most relevant parameters of the current problem are M, R, Ta, Pr and C_g.

To verify the accuracy of our numerical results, test computations were performed for steady Bénard–Marangoni convection in the case of no feedback control ($C_g = 0$). The M_c obtained in this case were identical to the results obtained by Namikawa et al. [33] as plotted in Fig. 2(a). It is observed that M_c decreases as R increases, showing that R is a destabilizing factor. Fig. 2(b) shows clearly the drastic stabilizing effect of C_g. It is interesting to note

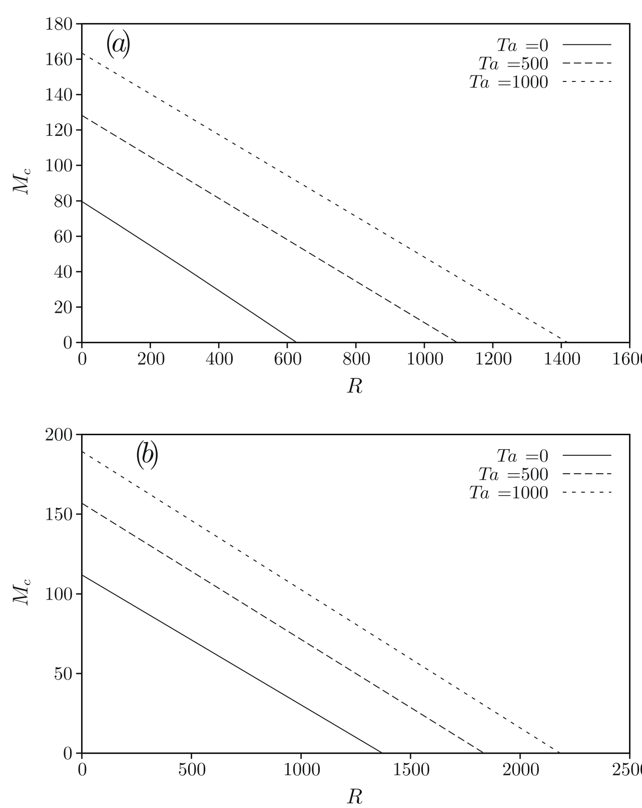

Fig. 2. M_c as a function of R for several values of Ta when (a) $C_g = 0$ and (b) $C_g = 2$.

that in the case without feedback control we need high rotation rates to stabilise the layer. However, with feedback control, convection can be delayed at low rotation rates of the layer. As has been shown by Chandrasekhar [2], the effect of increasing Ta is to inhibit the onset of steady convection.

Competition is possible between steady and oscillatory modes at the onset of convection as depicted in Fig. 3(a) and (b) for the case $C_g = 0$ and $C_g = 2$, respectively, with $Ta = 10,000$ and $R = 1,000$. The M_c at which the two modes of convection coexist are plotted in Fig. 4(a) as a function of R/R_c for several values of C_g for the case $Ta = 10,000$. Note that in this study R_c means the Rayleigh number for the case of no surface tension (i.e. $M = 0$). The corresponding a_c as a function of R/R_c for several values of C_g are presented in Fig. 4(b). As R/R_c goes to 1 the M_c decreases and a_c for oscillatory convection is always less than the a_c for steady convection as shown in Fig. 4. Fig. 5(a) shows the M_c at which the two modes of convection coexist as a function of Ta for several values of C_g when $R = 500$. Clearly, both rotation and feedback control are stabilizing. We observe that a_c on the steady curves are higher than that on the oscillatory curves. As Ta and C_g increase, the a_c at the onset of steady convection increases. However, the situation for oscillatory convection is slightly complicated: that is, there exists $Ta = \widehat{Ta}$, below which a_c at the onset of oscillatory convection increases with both Ta and C_g and above which a_c increases with Ta, but decreases with C_g.

An important situation that can be observed from Fig. 3 is that there exists Pr_c such that for all $Pr < Pr_c$, the M_c for oscillatory convection is always less than the M_c for steady convection. The Pr_c for the cases depicted in Figs. 3(a) and (b) are $Pr_c \approx 0.191$ and $Pr_c \approx 0.211$, respectively. The $Pr - Ta$ parameter space dividing stability domains into which either steady or oscillatory Marangoni convection is preferred has been presented by Sarma and Hashim [34] in the absence of feedback control. Sarma and Hashim's results for the specific case $C_g = 0 = R$ are shown in Fig. 6. Each curve in

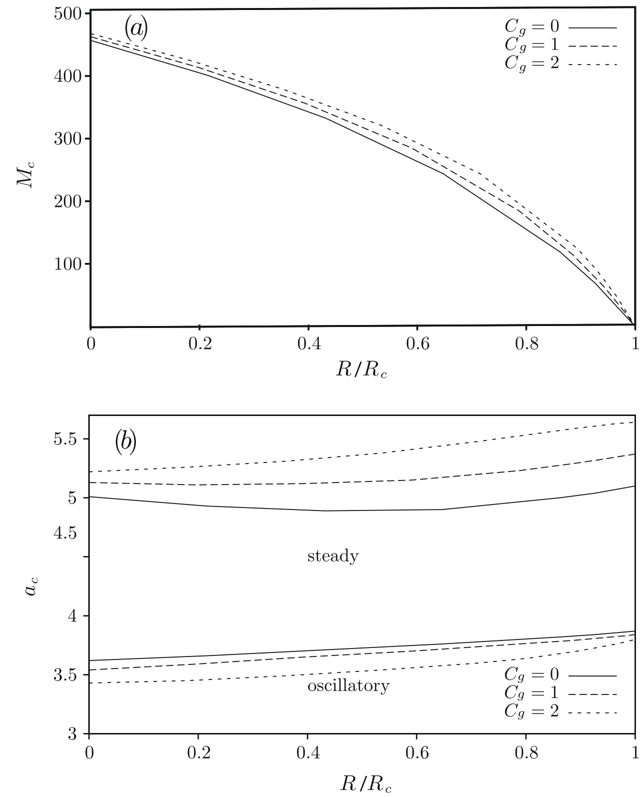

Fig. 4. (a) M_c and (b) a_c as functions of R/R_c with $Ta = 10,000$ for several values of C_g.

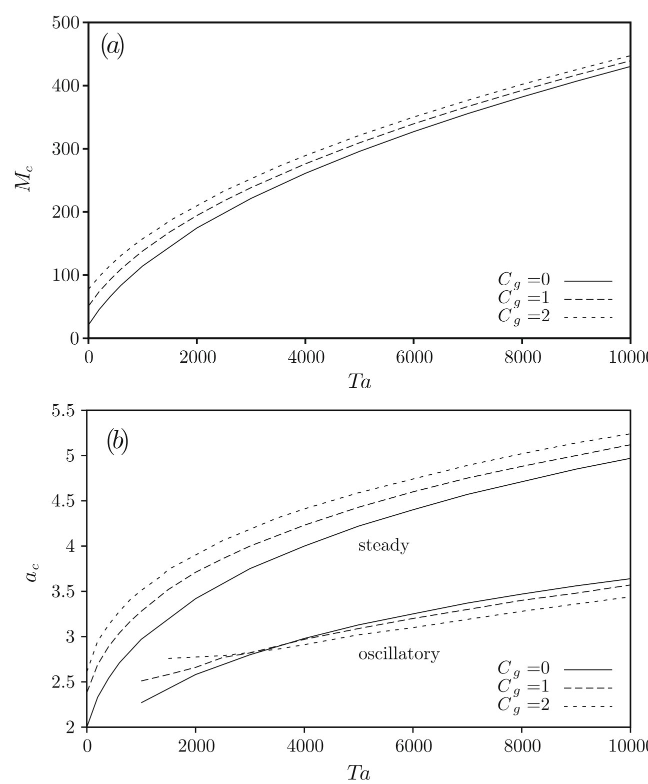

Fig. 5. (a) M_c and (b) a_c as functions of Ta for several values of C_g and $R = 500$.

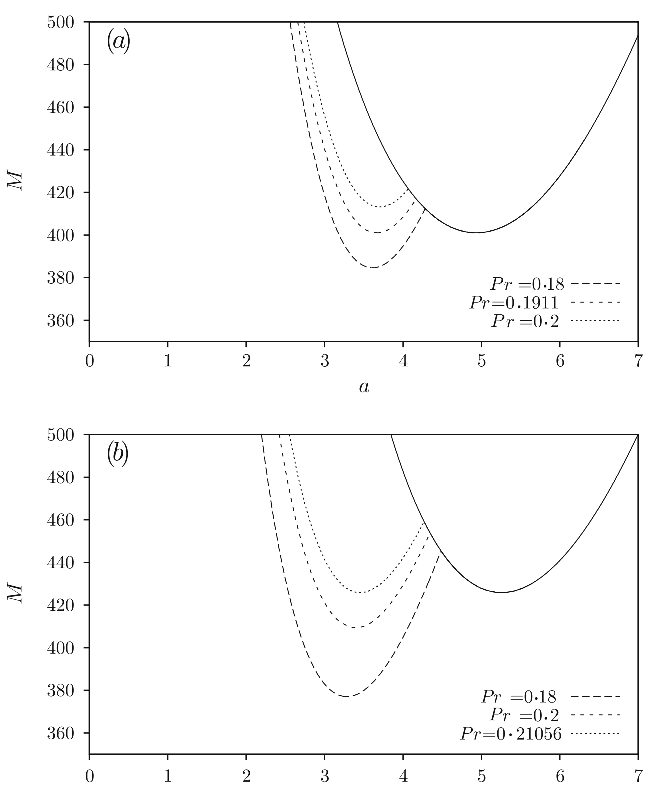

Fig. 3. Marginal stability curves for the onset of steady (solid) and oscillatory (dashed) convection for the case (a) $C_g = 0$ and (b) $C_g = 2$ with $Ta = 10,000$ and $R = 1,000$ for several values of Pr.

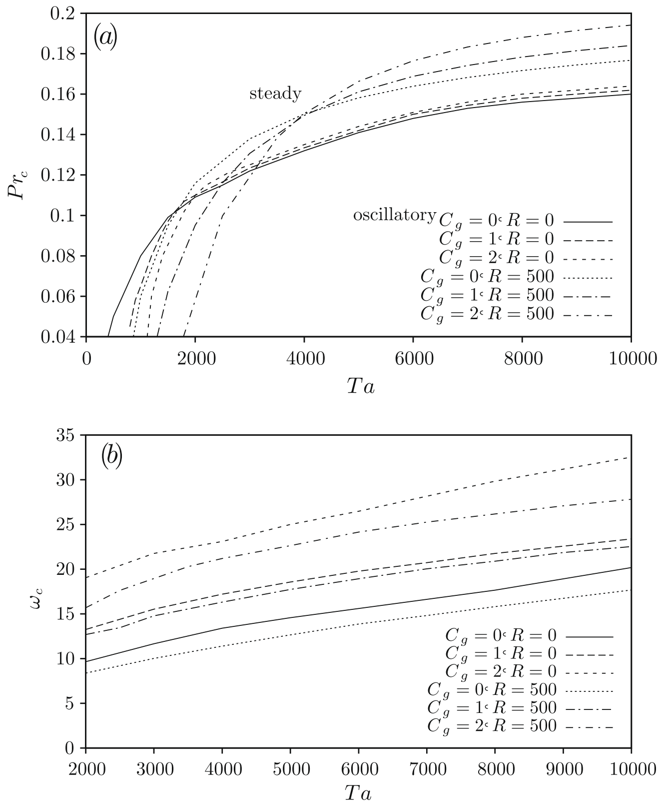

Fig. 6. (a) Pr_c and (b) ω_c as functions of Ta for several values of C_g.

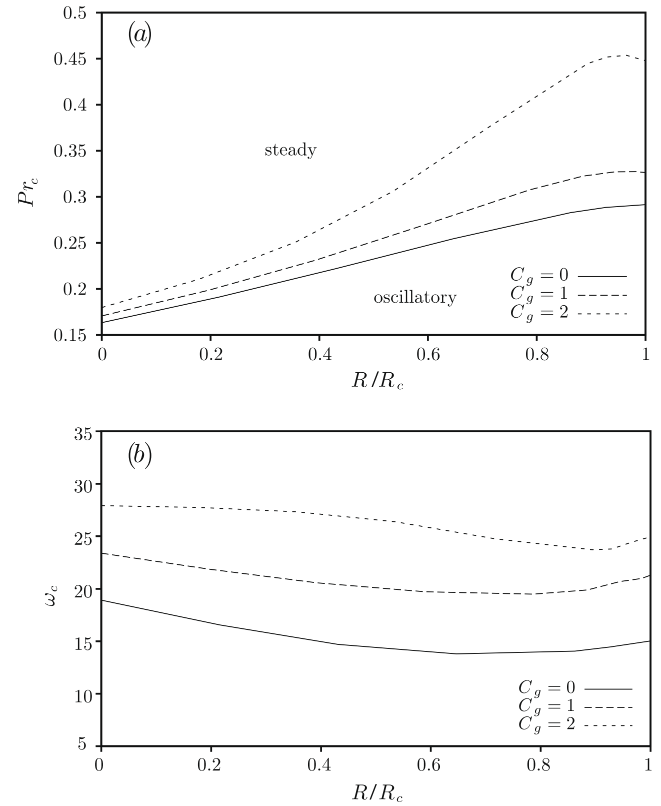

Fig. 7. (a) Pr_c and (b) ω_c as functions of R/R_c with $Ta = 10,000$ for several values of C_g.

Fig. 6(a) defines the boundary between the steady and oscillatory domains, i.e. points below the curve represent parameter combinations (Pr_c, Ta, C_g) for which convection sets in as oscillatory motions, while points above the curve are those for which steady convection is preferred. Note that the curves for the uncontrolled and controlled cases crisscross at certain values of Ta, say $Ta = \widetilde{Ta}$. If $Ta < \widetilde{Ta}$, the application of feedback control reduces the possibility of oscillatory convection. However, when $Ta > \widetilde{Ta}$ the reverse is true even for small controller gains, which is quite unexpected. We note that Bau [21] found the possibility of oscillatory convection for relatively large controller gains for the non-rotating layer. For a fixed C_g, the effect of increasing R is to shrink the region for oscillatory convection with moderate rotation, but the reverse is true for high rotation rates. The corresponding critical frequencies ω_c are shown in Fig. 6(b).

Now, as a final case with Ta fixed, we can determine the $Pr - R/R_c$ parameter space dividing stability domains into which either steady or oscillatory convection is preferred. In Fig. 7(a) we plot Pr_c as a function of R/R_c for several values of C_g. Clearly as shown in Fig. 7(a), Pr_c is, for the most part, an increasing function of R/R_c, i.e. as R/R_c increases, the region for oscillatory convection widens. The corresponding ω_c for the controlled case are always above the uncontrolled case as R/R_c is increased (see Fig. 7(b)).

6. Conclusions

Feedback control strategies to alter the flow patterns in Benard–Marangoni convection in a fluid layer heated from below and cooled from above were studied theoretically. It was demonstrated that the transition from the no-motion state to time-dependent convection can be significantly postponed. We have demonstrated that feedback control can be effectively used to stabilize the no-motion state of a fluid layer heated from below. More specifically, with the use of controller, one can increase the critical Marangoni number for the onset of convection. Rather than stabilizing an equilibrium state of the given system, the controller could be used to create new flow structures to optimize in such a way to suit particular process requirements, which was not explored in this study. In the case of no feedback control with the layer kept rotating, the critical Marangoni number can also be increased. It is demonstrated that generally the critical Marangoni number for transition from the no-motion to the motion state can be drastically increased by the combined effects of feedback control and rotation. In other words, for the case of no C_g, we need high rotation rate (Ta) to stabilize the layer, but in the presence of C_g, convection can be delayed at low rotation rates of the layer. We can conclude that, the combination of rotation and feedback control has a strong stabilizing effect. Also, we determined the influence of feedback control on the $R/R_c - Pr$ parameter space dividing stability domains into which either steady or oscillatory Bénard–Marangoni convection is preferred. Lastly, the controller described here can be used to provide active insulation and to suppress convection currents in material processing and crystal growth processes.

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